ISTITUTO DI ISTRUZIONE SUPERIORE
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MODULO CLIL
OPERATIONAL RESEARCH:
DECISION-MAKING PROBLEMS

DISCIPLINE COINVOLTE:

MATEMATICA

INGLESE

CLASSE 5C TURISMO **A.S. 2015/2016**

OPERATIONAL RESEARCH

- **It is the discipline of applying analytical methods to help making better decisions.**
- **It employs mathematical techniques to find optimal solutions to complex decision-making problems.**
- **Its birth is related to the studies that, in the years immediately preceding the outbreak of World War II, were conducted in Great Britain on the efficient use of radar as air defense.**
- **Its methods were employed to analyze strategical and tactical problems associated with military operations.**
- **In the decades after the war its techniques began to be applied to problems in business, industry and society.**

OPERATIONAL RESEARCH METHODOLOGY

Five steps of the Operational Research methodology:

1. **Identification of the problem.**
2. **Collection of data and analysis of the context.**
3. **Translation of the problem into a mathematical model.**
4. **Determination of the solution of the mathematical model.**
5. **Analysis and validation of results.**

A MATHEMATICAL MODEL FOR AN OPERATIONAL RESEARCH PROBLEM

It is a set of logical and mathematical relationships that represents aspects of the situation under study in a clear, precise and simplified way. It consists of:

$$y = f(x_1, x_2, \ldots, x_n)$$

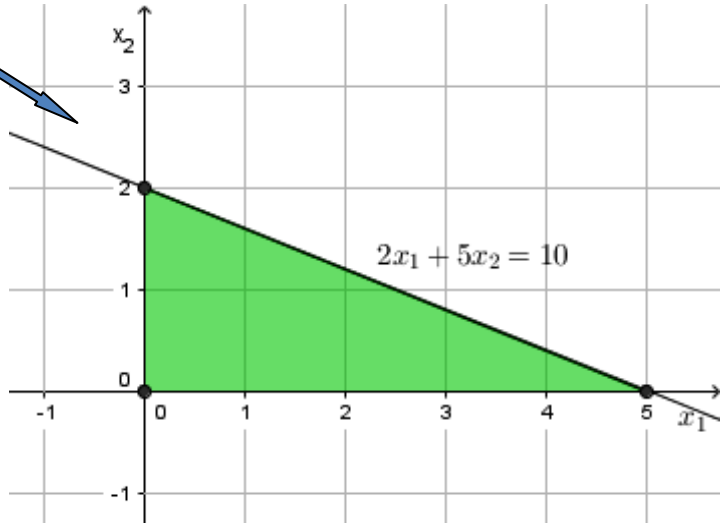
an objective function of several decision variables

a feasible region mathematically expressed by constraints such as

$$\begin{cases} x_1 \geq 0 \\ x_2 \geq 0 \\ 2x_1 + 5x_2 \leq 10 \end{cases}$$

nonnegative constraints

technical constraints



DIFFERENT TYPES OF DECISION-MAKING PROBLEMS IN ECONOMICS

- **Discrete decision-making problems.**
- **Continuous decision-making problems.**

- **Decision-making problems under uncertainty.**
- **Decision-making problems under certainty.**

- **Short-run decision-making problems.**
- **Long-run decision-making problems.**

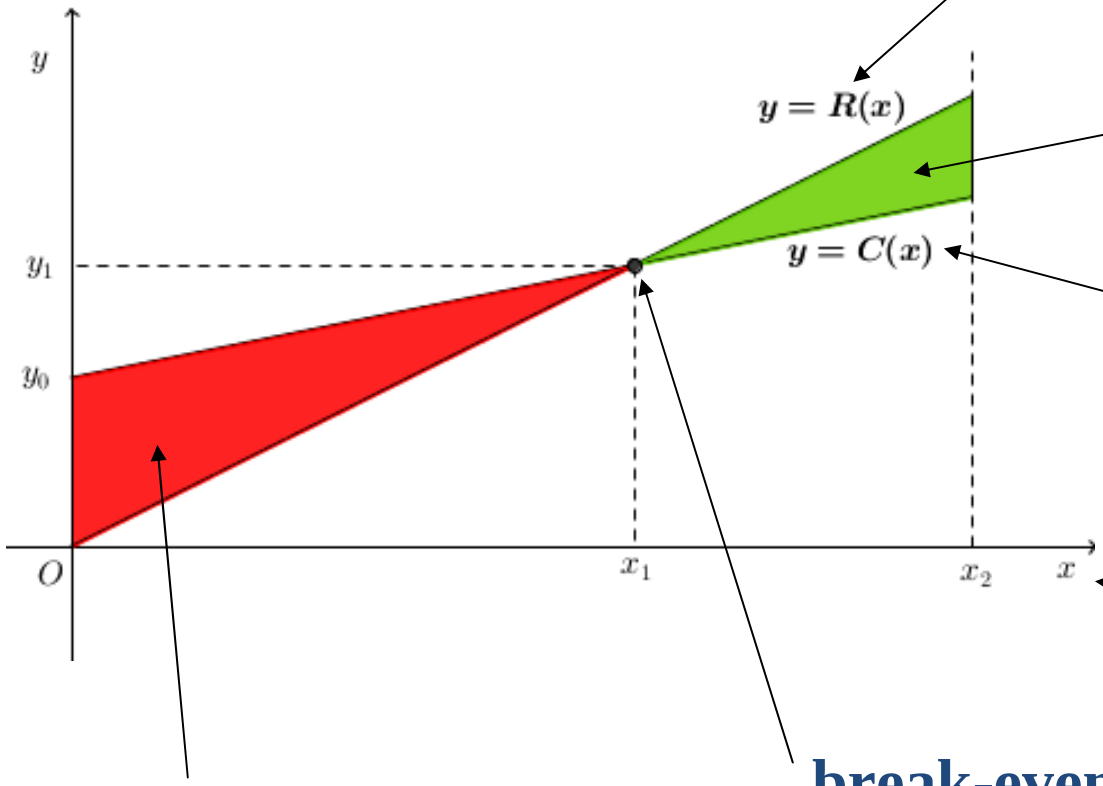
MATHEMATICS AND ECONOMICS

- **Fixed costs are costs that do not change when the quantity of output changes (e.g. rent, costs for equipment maintenance)**
- **Variable costs are costs that vary with output (e.g. costs for the material consumed during production, hourly wages)**
- **Total cost: fixed costs + variable costs.**

- **Total revenue is the total amount of a company's sales and other sources of income.**

- **Profit: total revenue – total cost.**

Total revenue interacts with total cost in determining the level of output at which the firm achieves its objective of profit maximization.



total revenue function

profit area

total cost function

output

loss area

break-even point: it is the point where the gains equal the losses

SOME EXAMPLES OF SHORT-RUN DECISION-MAKING PROBLEMS UNDER CERTAINTY

A continuous decision-making problem

Problem 1. **A company can produce a maximum of 3500 kilograms of a certain type of product daily. For each kilogram produced the cost of manufacturing and raw materials is € 0.12 and the daily fixed costs are € 200. Find the maximum profit and the minimum amount so as not to have a loss, knowing that each kilogram is sold at € 0.46.**

Solution.

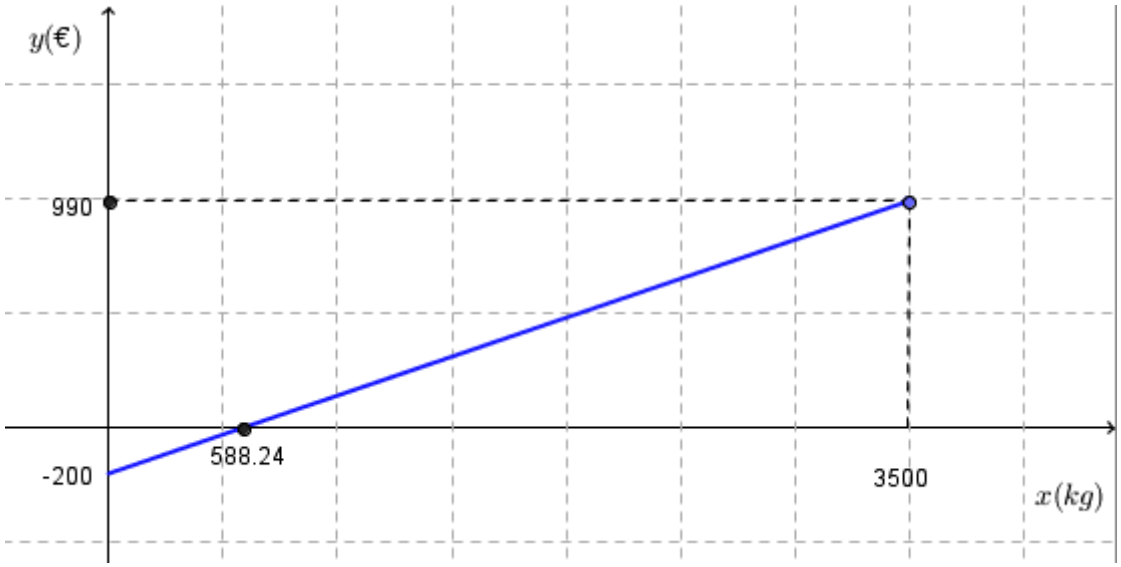
x = **number of kilograms produced and sold**

Mathematical model:

find the maximum of the function

$y = 0.34x - 200$

$0 \leq x \leq 3500$



Answer:

The maximum profit is € 990 and it corresponds to a quantity of 3500 kilograms. The minimum amount so as not to have a loss is 588.24 kilograms.

The objective function is expressed by two functions

Problem 2. **A business owner buys an amount of fertilizer from a producer. The cost is € 15 per hundredweight for purchases up to 100 hundredweights and € 10 per hundredweight for the hundredweights exceeding the 100 hundredweights. The producer resells the fertilizer at € 30 per hundredweight. For this activity the producer has a fixed cost of € 500. Knowing that he can sell a maximum of 300 hundredweights each month, find out the number of hundredweights he should buy and sell monthly in order to maximize the profit.**

(English translation of problem number 4 on page 236, A. Gambotto Manzone, B. Consolini, "Matematica con applicazioni informatiche 3", Tramontana.)

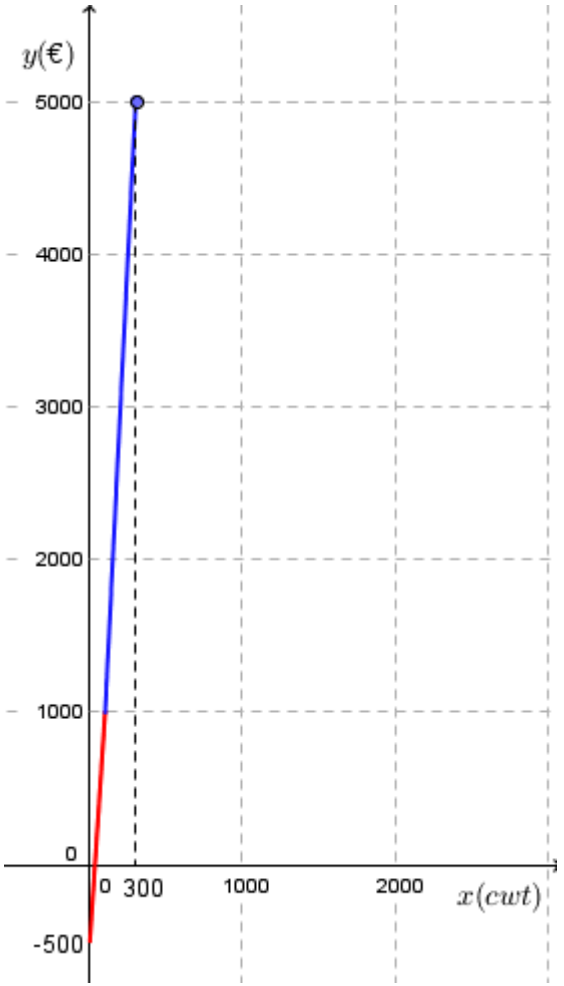
Solution.

x = **number of hundredweights of fertilizer to buy and sell**

Mathematical model:

find the maximum of the function

$$y = \begin{cases} 15x - 500 & \text{if} \quad 0 \le x \le 100 \\ 20x - 1000 & \text{if} \quad 100 < x \le 300 \end{cases}$$



Answer:

The maximum profit is € 5000 and it is obtained by buying and reselling 300 hundredweights of fertilizer per month.

The objective function is expressed by two functions

Problem 3. **A seller buys a product from a producer and then resells it to retailers. The cost of the product is €10 per kilo for monthly purchases up to 300 kilograms and it is € 8 per kilo if the monthly purchases exceed 300 kilograms. The fixed monthly costs are € 630. The seller sells the product at € 22 per kilo. Knowing that he can buy a maximum of 600 kilograms per month, find the maximum profit and the minimum amount so as not to have a loss.**

(English translation of problem number 31 on page 615, A. Gambotto Manzone, B. Consolini, "Matematica con applicazioni informatiche 3", Tramontana.)

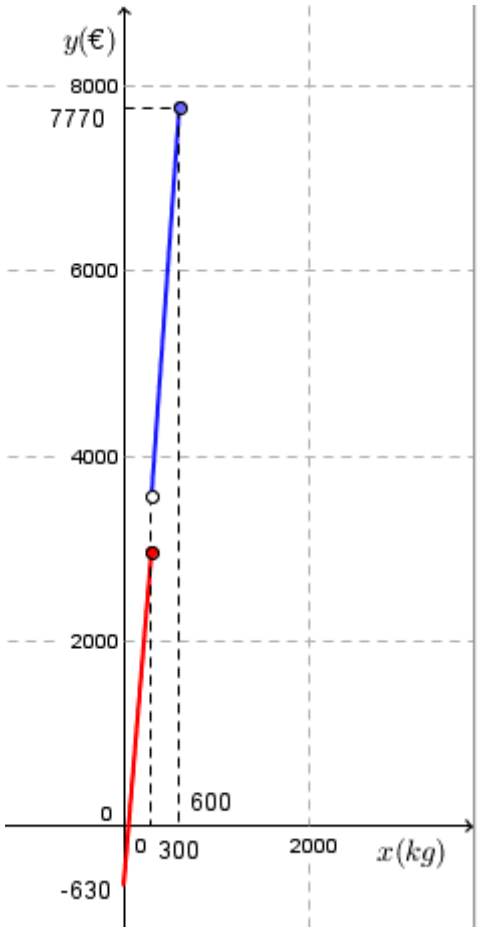
Solution.

x = **number of kilograms of product to buy and sell**

Mathematical model:

find the maximum of the function

$$y = \begin{cases} 12x - 630 & \text{if} \quad 0 \le x \le 300 \\ 14x - 630 & \text{if} \quad 300 < x \le 600 \end{cases}$$



Answer:

The maximum profit is € 7770 and it is obtained by buying and reselling 600 kilograms of product per month.

The minimum amount so as not to have a loss is 52,5 kilograms.

The choice of the best option

Problem 4. **In order to transport their products a forwarding company can choose between two different clients that apply the following conditions:**

Client A: a flat rate of € 115 and 0.25 per hundreweight of transported goods;

Client B: a flat rate of € 58 and 1.2 per hundredweight of transported goods.

Find the most convenient solution for the forwarding company according to the quantity transported, knowing that it is not possible to transport more than 10 tons monthly.

(English translation of problem number 80 on page 1227, M. Bergamini, A. Trifone, G. Barozzi, “Matematica.rosso”, Zanichelli.)

Solution.

x = **number of hundredweights of transported goods**

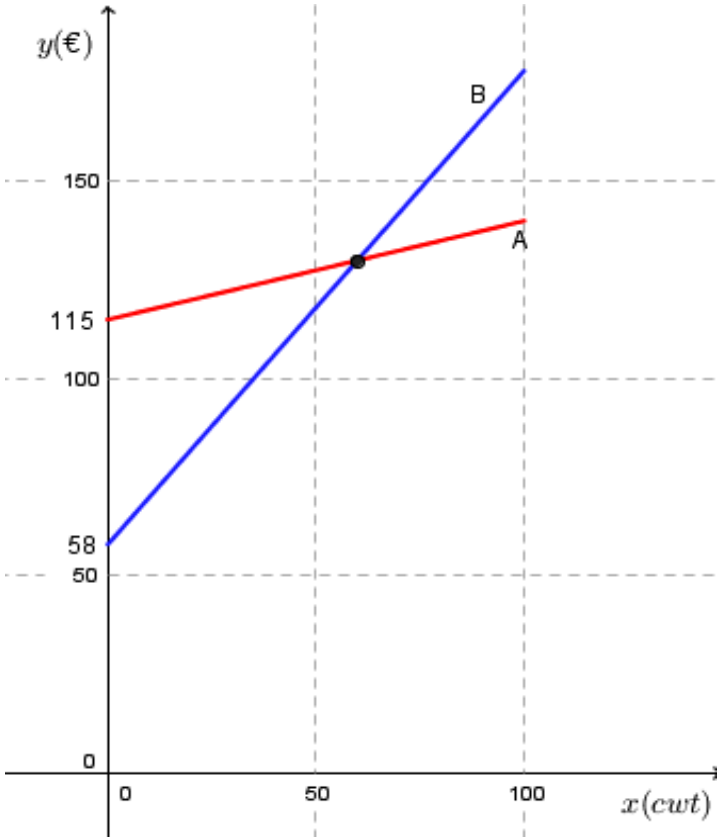
Mathematical model:

find the values of x for which one of the following two functions assumes greater values than the other one

$y_A = 115 + 0.25x$

$0 \le x \le 100$

$y_B = 58 + 1.2x$



Answer:

If $0 \le x < 60$ **alternative A is more convenient**

If $60 < x \le 100$ **alternative B is more convenient**

If $x = 60$ **both A and B are convenient**